Restructuring Problems

• Some aspects of problems are not solved through a gradual search process. The problem may be solved suddenly by 'seeing' the problem differently

• solutions may require insight based on a restructuring of the problem.

The Handcuffs Puzzle

The Set-Up For this puzzle you need two people, some rope and some empty space to do the puzzle in. Each person will need a piece of rope with a loop tied in both ends, so it can be worn as handcuffs. The rope should be reasonably long, so that the person wearing it can easily step over it if they want.

Each person puts on a complete set of handcuffs. Before putting them on, they loop their handcuffs around each other so they are tied together. Each person should wear a complete set of handcuffs. They then have to get themselves apart while following these rules:

- The handcuffs cannot be removed.
- Do not break, cut, saw through, bite through or in any other way damage the rope. Damaging each other is probably a bad idea too.

content copied from: http://ccins.camosun.bc.ca/~jbritton/jbhandcuff.htm
True Story

A professor comes to a University to give a talk. They set up a slide projector for him, but neglect to test it. During the talk it becomes apparent that the projector is set too low. Graduate students and professors gather around the projector trying to fix the problem. They call out for a book to raise the front end of the projector. The book turns out to be too thick. They call out for a thinner book... While everyone is searching for another book, one person comes over and quietly solves the problem...

**Punchline.** All these academics in one room, and nobody knows how to *open* a book??

(from Ashcroft, 2002)

Another problem

- A man climbs a mountain on Sat, leaving at daybreak and arriving at the top at sundown. He spends the night, gets up the next day at daybreak and heads down the mountain, following exactly the same path he climbed the day before.
- Question: will there be any time during the second day when he will be at exactly the same point on the mountain he was at that time the first day?

Gestalt View of Problem Solving

**Unit of thought:** Organization, Mental Structures

**Mental Activity:** Reorganization, Insight, Productive Thinking

**Structural understanding:** the ability to comprehend how the problem elements fit together to form a structure in order to solve a problem.
Kohler (1925): monkey and banana problem.

Chimpanzees appeared to have an insight into the problem before solving it.

Kohler’s (1956) two-stick problem

Six stick problem

With these six sticks:
\[ \begin{array}{cccc}
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\end{array} \]

Wrong solution:

Answer:

Make four of these:

\[ \begin{array}{c}
\_ \\
\_ \\
\_ \\
\end{array} \]

\[ \begin{array}{c}
\_ \\
\_ \\
\_ \\
\end{array} \]

Answer:
Two Kinds of Thinking

- Productive vs. reproductive
- Insight vs. trial and error
- Structural understanding vs. rote memory
- Ill-defined vs. well-defined

Wertheimer’s Parallelogram problem

\[
\text{Area} = \text{b} \times \text{h}
\]

Understanding Method

Rote Method:

\[
\text{Area} = \text{b} \times \text{h}
\]

Transfer Problems:

Katona’s Matchstick Problem

The Problem:
Given: five matchsticks arranged to form a square. Move three sticks to form four squares.

Rules:
1. Only use whole matchsticks.
2. Matches cannot be broken or cut.
3. Matches cannot overlap.

Results:

<table>
<thead>
<tr>
<th>Group</th>
<th>Initial Sticks</th>
<th>New Sticks</th>
<th>Test After 1 Week</th>
<th>Test After 3 Weeks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Women</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

Note: The students who passed the test after 3 weeks were a different group from those who passed after 1 week.

From Katona (1968)
Insight

• Gestalt concept of insight: moments of sudden comprehension. Gestaltists argued it might result from:
  a) extended unconscious leaps in thinking
  b) greatly accelerated mental processing
  c) restructuring elements of problem to reveal new possibilities

Gestaltists were vague on these processes.

• Others have argued that insight is nothing special -- Insights are merely significant products of ordinary thinking processes

(e.g. Perline, 81; Weisberg, 1986,1995; Langley, Simon, Bradshaw & Zytkow, 1988)

Evidence for concept of insight: Metcalfe experiment

1) Noninsight problem (algebra):
   factor $16y^2 - 40yz + 25z^2$

2) Insight problem (nonroutine):
   A prisoner was attempting escape from a tower. He found in his cell a rope which was half long enough to permit him to reach the ground safely. He divided the rope in half and tied the two parts together and escaped. How could he have done this?

Results (1)

• First result: subjects “feelings of knowing” (beforehand) only predicted eventual success of solving the problem for noninsight problems.

• At 15 seconds intervals, ss. rated how close they felt to solving the problem:
   1=cold (nowhere close to solution)
   ....
   7=hot (problem is virtually solved)
Different Interpretations of Insight Problems

• Insights problems really do involve sudden moments of comprehension

OR

• It just appears that way. Solvers might be constantly on the path towards a solution but they do not recognize they are getting close

Rigidity in problem solving: Functional Fixedness

Maier’s (1931) two-string problem
Only 39% of subjects were able to see solution within 10 minutes.

Duncker’s problem: support a candle on a door

A box of tacks, some matches, and a candle

Hat Rack Problem
Why people get stuck solving problems

Functional Fixedness:

Subjects who utilize an object for a particular function will have more trouble in a problem-solving situation that requires a new and dissimilar function for the object.

Mental set -- Einstellung

A frame of mind involving a particular way of representing problem or solving a problem

Luchins Water Jar Problems

How would you use 3 jars A, B, and C with the indicated capacities to measure out the desired amount of water?

<table>
<thead>
<tr>
<th>Example</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>DESIRED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>29</td>
<td>3</td>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

Solution A-3B
Luchins Water Jar Problems

How would you use 3 jars with the indicated capacities to measure out the desired amount of water?

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>DESIRED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1</td>
<td>3</td>
<td>77</td>
<td>21</td>
<td>50</td>
</tr>
<tr>
<td>Problem 2</td>
<td>8</td>
<td>80</td>
<td>4</td>
<td>60</td>
</tr>
<tr>
<td>Problem 3</td>
<td>5</td>
<td>43</td>
<td>12</td>
<td>21</td>
</tr>
<tr>
<td>Problem 4</td>
<td>6</td>
<td>42</td>
<td>9</td>
<td>21</td>
</tr>
<tr>
<td>Problem 5</td>
<td>3</td>
<td>49</td>
<td>23</td>
<td>20</td>
</tr>
<tr>
<td>Problem 6</td>
<td>4</td>
<td>48</td>
<td>18</td>
<td>22</td>
</tr>
</tbody>
</table>

A simpler solution exists for problems 5 and 6, but subjects reached a state of “Einstellung” where they kept applying old successful problem solving methods.

Luchins Results

<table>
<thead>
<tr>
<th>Group</th>
<th>Einstellung Solution (percent)</th>
<th>Direct Solution (percent)</th>
<th>No Solution (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>0</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>Experimental</td>
<td>74</td>
<td>26</td>
<td>0</td>
</tr>
</tbody>
</table>

(Control subjects did not have the first four problems)
But...

Are functional fixedness, mental sets, etc. really a problem?

- Generalization from experience leads to efficiency
- How often do mental sets and functional fixedness save time and computation?
Silveira’s (1971) incubation experiment

- You are given four separate pieces of chain that are each three links in length. It costs 2c to open a link and 3c to close a link. All links are closed at the beginning of the problem. Your goal is to join all 12 links of chain into a single circle at a cost of no more than 15c.

Results

- Control group: worked for half hour
  \[ \rightarrow 55\% \text{ solved it} \]
- Experimental group 1: worked half hour interrupted by half hour break
  \[ \rightarrow 64\% \text{ solved it} \]
- Experimental group 2: worked half hour interrupted by four hour break:
  \[ \rightarrow 85\% \text{ solved it} \]

Incubation

- Time away from a problem provides new insights, or otherwise facilitates, the problem solving process. How does this work?
  1) release from a problem solving set, or functional fixedness
  2) retrieval of new information by changing context
  3) recovery from fatigue conscious problem solving in the interim

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Start at Phil's house. At first, you can only make right turns through the maze. Each time you cross the red zigzag sign (under Carl's auto repair), the direction in which you turn changes. So, after the first time you cross that sign, you can then only make left turns; after the second time, you switch back to right turns only, etc. How can Carl's auto repair be reached?