

Inductive Reasoning
&
Hypothesis Testing

Logical Reasoning and Human Nature

- Historically, many researchers believed that logical reasoning is an essential part of human nature
 - Aristotle
 - Boole (1854). A book on logical calculus
“An investigation of the laws of thought”
 - Rational behavior = logical thinking
(deductive reasoning)
- However, humans are not natural logical reasoners

Deductively valid?

Premise: All cars have wheels

Premise: All wheels are round

Conclusion: All cars have round wheels

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Premise: I have a diamond

Premise: Most diamonds are shiny

Conclusion: My diamond is shiny

Premise: John is 93

Conclusion: John will not do a double back flip today

Inductive vs. Deductive Reasoning

- **Deductive reasoning:**
 - conclusion follows logically from premises
- **Inductive reasoning:**
 - conclusion is likely based on premises.
 - involves a degree of uncertainty
- Most reasoning in real-world is based on induction

Inductive Reasoning

- Reason from **observable** information to **unobservable** and **uncertain** information
- E.g. concept learning with Google Sets

<http://labs.google.com/sets>

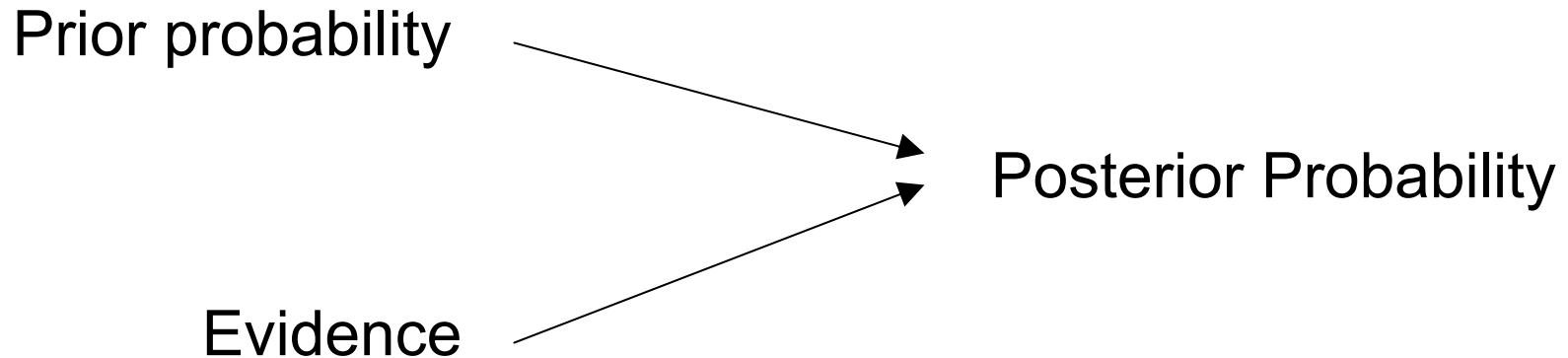
Real world inductive inferences

- Medical diagnosis:
 - Symptoms, test outcomes (observable) → Diseases (unobservable)
- Scientific reasoning:
 - Experimental data (observable) → Hypotheses (unobservable)
- Law:
 - facts (observable) → guilt (unobservable, uncertain)

Reasoning under Uncertainty

- **Bayes rule** tells us how to optimally reason with uncertainty.
- Allows us to say how we believe something to be true based on **prior beliefs** and **new available evidence**

Bayes Rule



Bayes rule tells us how the available evidence should alter our belief in something being true

Do people reason like Bayes rule?

- Problems understanding Conditional probability
 - Doctors need to calculate the probability of disease given the observed symptoms: $P(\text{disease} \mid \text{symptoms})$
 - Sometimes $P(\text{symptoms} \mid \text{disease})$ is used incorrectly when reasoning about the likelihood of a disease
- Why is this wrong?

The base rate is important

- To get $P(\text{disease} \mid \text{symptom})$, you need to know about $P(\text{symptom} \mid \text{disease})$ and also the **base rate** -- prevalence of the disease before you have seen patient
- More intuitive example:
 - what is the probability of being tall given you are player in the NBA?
 - what is the probability of being a player in the NBA given that you are tall?

$$P(\text{NBA player} \mid \text{tall}) \neq P(\text{tall} \mid \text{NBA player})$$

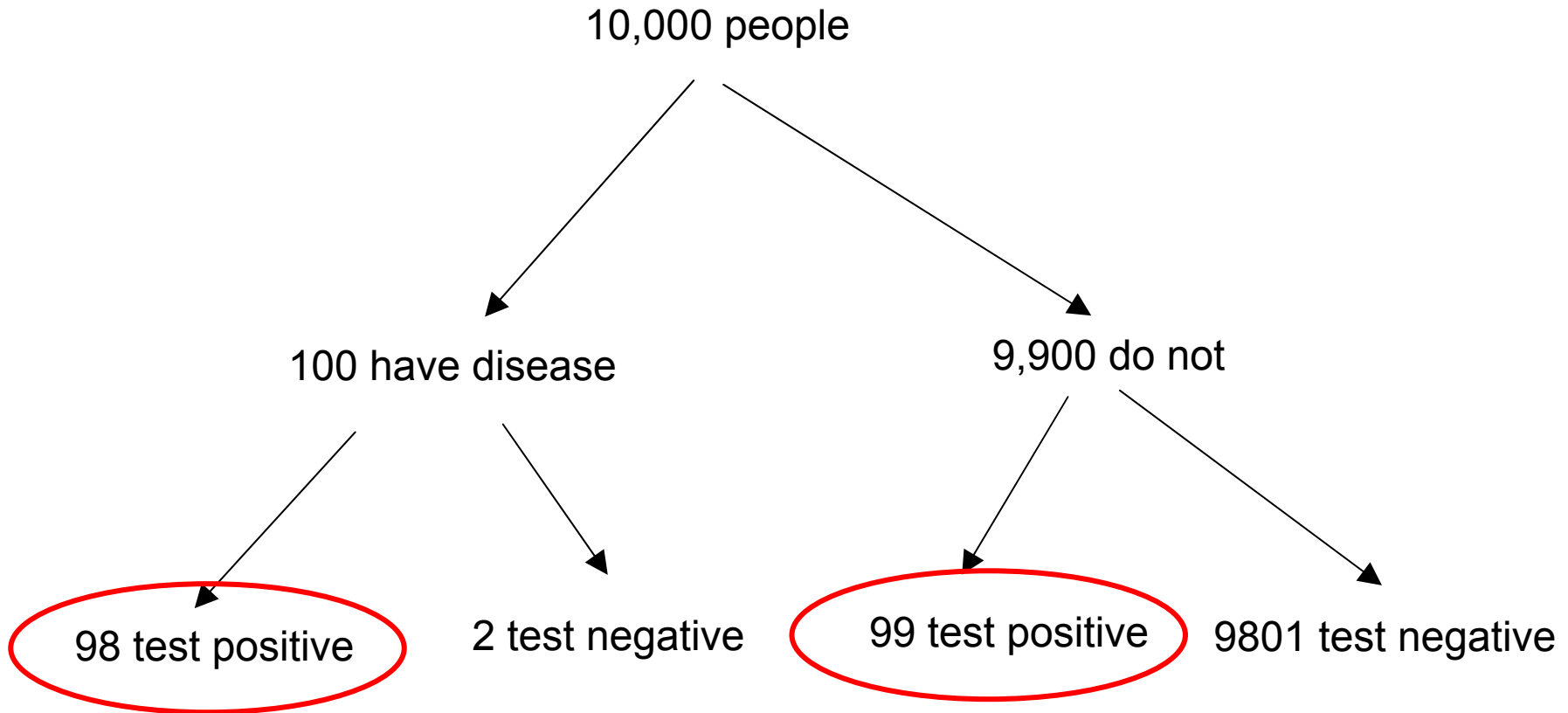
Reasoning with base rates

- Suppose there is a disease that affects 1 out of 100 people
- There is a diagnostic test with the following properties:
 - If the person has the disease, the test will be positive 98% of the time
 - if the person does not have the disease, the test will be positive 1% of the time
- A person tests positive, what is the probability that this person has the disease?
 - Frequent answer =
 - Correct answer \approx

Are we really that bad in judging probabilities?

According to some researchers (e.g., Gigerenzer), it matters **how** the information is presented and processed. Processing **frequencies** is more intuitive than **probabilities**

A counting heuristic (in tree form)



$$P(\text{disease} \mid \text{test positive}) = 98 / (98 + 99) \approx .50$$

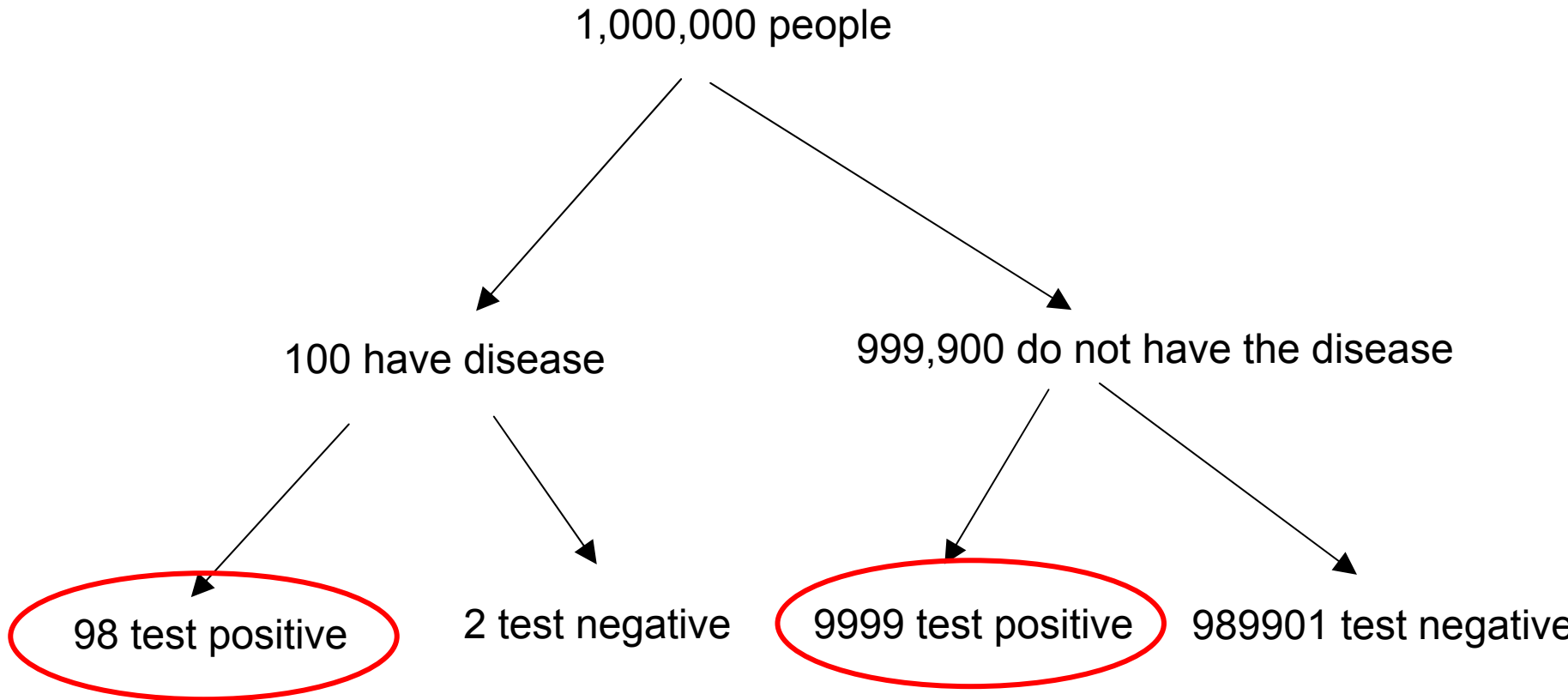
The same thing in words ...

- Let's take 10,000 people.
- On average, 100 out of 10,000 actually have the disease and 98 of those will test positive (98% true positive rate)
- Among the 9,900 who do not have the disease, the test will falsely identify 1% as having it. 1% of 9,900 = 99
- On average, out of 10,000 people:
98 test positive and they have the disease
99 test positive and they do not have the disease.
- Therefore, a positive test outcome implies a $98/(98+99) \approx 50\%$ chance of having the disease

Change the example

- What now if the disease affects only **1 out of 10,000** people?
- Assume same diagnosticity of test (98% true positive rate, 1% false positive rate)
- A person tests positive, what now is the probability that this person has the disease?

A counting heuristic (in tree form)



$$P(\text{disease} \mid \text{test positive}) = 98 / (98 + 9999) = .0097 \quad (\text{smaller than } 1\%)$$

Bayes Rule

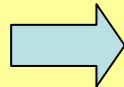
- The previous example essentially is a simple way to apply Bayes rule:

$$P(\text{disease} \mid \text{positive}) = \frac{P(\text{positive} \mid \text{disease})P(\text{disease})}{P(\text{positive} \mid \text{disease})P(\text{disease}) + P(\text{positive} \mid \text{not disease})P(\text{not disease})}$$

$$P(\text{positive} \mid \text{disease}) = .98$$

$$P(\text{positive} \mid \text{not disease}) = .01$$

$$P(\text{disease}) = .0001$$



$$P(\text{disease} \mid \text{positive}) = .0097$$

Normative Model

- Bayes rule tells you how you should reason with probabilities – it is a prescriptive (i.e., **normative**) model
- But do people reason like Bayes?
In certain circumstances, the base rates are neglected
base rate neglect

The Taxi Problem: version 1

- A witness sees a crime involving a taxi in Carborough. The witness says that the taxi is **blue**. It is known from previous research that witnesses are correct **80%** of the time when making such statements.
- What is the probability that a blue taxi was involved in the crime?

The Taxi Problem: version 2

- A witness sees a crime involving a taxi in Carborough. The witness says that the taxi is **blue**. It is known from previous research that witnesses are correct **80%** of the time when making such statements.
- The police also know that 15% of the taxis in Carborough are blue, the other 85% being green.
- What is the probability that a blue taxi was involved in the crime?

Base Rate Neglect: The Taxi Problem

- Failure to take prior probabilities (i.e., base rates) into account
- In the taxi story, the addition of:

“The police also know that 15% of the taxis in Carborough are blue, the other 85% being green.”

has little influence on rated probability

Base Rate Neglect (2)

- Kahneman & Tversky (1973).

group A: 70 engineers and 30 lawyers

group B: 30 engineers and 70 lawyers

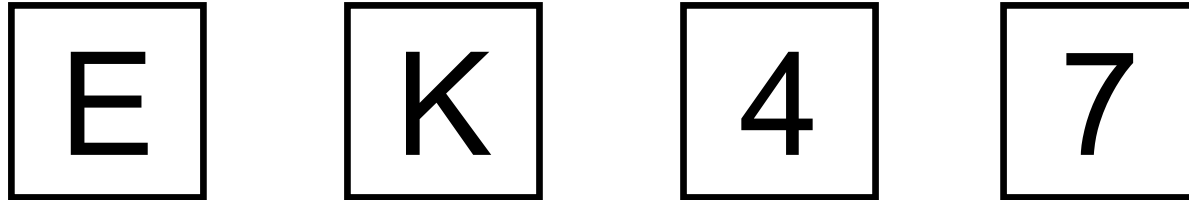
- What is probability of picking an engineer in group A and B?
Subjects can do this ...

Provide some evidence ...

- “Jack is a 45 year-old man. He is married and has four children. He is generally conservative, careful, and ambitious. He shows no interest in political and social issues and spends most of his free time on his many hobbies, which include home carpentry, sailing, and mathematical puzzles”
- What now is probability Jack is an engineer?
- Estimates for both group A and group B was $P = .9$

Hypothesis Testing

Wason Selection Task



“If a card has a vowel on one side, then it has an even number on the other side”

Which cards do you need to turn over to test the correctness of the rule?

Concrete examples are much easier

- If a person is drinking beer, then the person must be over 21. How to test whether somebody is abiding by this rule?

Drinking
beer

Drinking
Coke

16 years
of age

22 years
of age

Result: 74% answered correctly

Conclusion from Wason Selection Task

- From a pure deductive point of view, subjects fail to reason appropriately
- However, from an inductive point of view, subjects' choices are quite reasonable *under certain assumptions*:
 - Rules such as “If **Cause** then **Effect**” are interpreted *probabilistically*
 - Causes are rare
 - Effects are rare

Hypothesis Testing

- 2-4-8 is a set of numbers that conforms to a rule.
- **Discover the rule** by querying with any set of three numbers and I'll give feedback whether it is a *positive* or *negative* example.

Confirmation Bias

- Wason (1960): subjects test hypotheses by generating positive rather than negative examples
- Popper (1959): confirmatory strategies provide ambiguous information. The hypothesis may be correct or another hypothesis may be correct → scientists should try to falsify their theories
- However, in many cases, it might make more sense to confirm hypotheses, and not to attempt falsification



Interaction



