Thinking part III
Problem Solving
Solve this maze at your leisure.

Start at phil’s house. At first, you can only make right turns through the maze. Each time you cross the red zigzag sign (under Carl’s auto repair), the direction in which you turn changes. So, after the first time you cross that sign, you can then only make left turns; after the second time, you switch back to right turns only, etc. How can Carl’s auto repair be reached?
Overview

• Well-defined problems
  – heuristics to search problem spaces

• Ill-defined problems
  – obstacles in problem-solving
  – mental set/functional fixedness

• Insight vs. trial & error

• Expert problem solving
  – role of practice
  – talent
Views of Problem solving

• **Well-defined problems**
  – Much studied in AI
  – Requires search
  – Domain general heuristics for solving problems

• What about **ill-defined** problems?
  – No real mechanisms for dealing with these
  – The problem may be solved suddenly by ‘seeing’ the problem differently
  – Often requires developing a **suitable representation**
Problem solving as search

Play the game: http://vornlocher.de/tower.html
Problem Solving is a search problem

Search Space

Solution

Initial state

Goal state
Search spaces can be large

<table>
<thead>
<tr>
<th>#DISCS</th>
<th>#STATES</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$3^3 = 27$</td>
</tr>
<tr>
<td>4</td>
<td>$3^4 = 81$</td>
</tr>
<tr>
<td>5</td>
<td>$3^5 = 243$</td>
</tr>
<tr>
<td>6</td>
<td>$3^6 = 729$</td>
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</tbody>
</table>

**FIGURE 3**  A search space for the Tower of Hanoi.
What if the search space is too large?

• It is not possible to enumerate the entire search space for many well-defined problems.

• We must use **heuristics**
  – Not guaranteed to work but easy to implement
  – Example heuristics
    • **Trial and error**
    • **Hill climbing**
    • **Means-end analysis**
Trial and Error

- Edward L. Thorndike (1874-1949) found that many animals search by trial and error.
- Found that cats in a “puzzle box” (see left) initially behaved impulsively and apparently random.
- After many trials in puzzle box, solution time decreases.

In order to escape the animal has to perform three different actions: press a pedal, pull on a string, and push a bar up or down.
Hill Climbing

• Find some measure of the distance between your present state and the end state.
  – Take a step in the direction that most reduces that distance
## Hill Climbing

- Might lead to suboptimal solutions: **local maximum**

### Diagram

A diagram illustrating the concept of hill climbing, showing a grid with a marked food source and a path that leads to a local maximum.

```
<table>
<thead>
<tr>
<th>3</th>
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<th>5</th>
<th>FOOD</th>
<th>5</th>
<th>4</th>
<th>3</th>
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<td>2</td>
<td>3</td>
<td></td>
<td>2</td>
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<td>0</td>
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<tr>
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</tbody>
</table>
```

The grid represents a path with a fence and a food source at the center, illustrating the concept of hill climbing and the challenge of reaching the local maximum.
Means-end analysis

• Set up a goal
• Look for a difference between current state and goal or subgoal state
• Find an **operator** to reduce this difference. One operator is the setting of a new **subgoal**
• Apply operator
• Repeat until final goal is achieved
Setting subgoals in means-end analysis

- Painting your house (GOAL 1)
- Apply paint (SUBGOAL 2)
- Need paint and brush (SUBGOAL 3)
- Go to hardware store (SUBGOAL 4)
  - Went to hardware store (SUBGOAL 4)
  - Got paint and brush (SUBGOAL 3)
  - Apply paint (SUBGOAL 2)
  - Paint the house (GOAL 1)
What about ill-defined problems?

• No real mechanisms for dealing with these

• According to Gestalt psychologists, the problem may be solved suddenly by ‘seeing’ the problem differently

• Often requires developing a suitable representation
Possible or Impossible?

Starting in the square marked by the circle, draw a line through all the squares without picking up your pencil, without passing through a square more than once, without diagonal lines and without leaving the checkerboard.
Six stick problem

With these six sticks:

Wrong solution:

Make four equilateral triangles:

Answer:
Functional Fixedness

Maier’s (1931) two-string problem
Only 39% of subjects were able to see solution within 10 minutes.
Why people get stuck solving problems

*Functional Fixedness:*

Subjects who utilize an object for a particular function will have more trouble in a problem-solving situation that requires a new and dissimilar function for the object.

*Mental set:*

A frame of mind involving a particular way of representing problem or solving a problem.
Exercise: Assume that a steel pipe is imbedded in the concrete floor of a bare room as shown below. The inside diameter is .06" larger than the diameter of a ping-pong ball (1.50") that is resting gently at the bottom of the pipe. You are one of a group of six people in the room, along with the following objects:

- 100' of clothesline
- A carpenter’s hammer
- A chisel
- A box of Wheaties
- A file
- A wire coat hanger
- A monkey wrench
- A light bulb

List as many ways you can think of (in five minutes) to get the ball out of the pipe without damaging the ball, tube, or floor.

Insight

- Seemingly sudden understanding of a problem

- Often involves conceptualizing a problem in a totally different way (e.g. six stick problem, overcoming functional fixedness)

- How can we distinguish between problems requiring insight and problems requiring noninsightful problem solving?
Kohler (1945): monkey and banana problem.

Kohler claimed that chimpanzees appeared to have an insight into the problem before solving it; he claimed there was no process of trial-and-error.
Video: insight or trial and error/ hillclimbing? (1 min)

http://www.youtube.com/watch?v=mDntbGRPeEU&feature=related
Possible evidence for concept of insight:
Metcalfe and Weibe (1987) experiment

1) Noninsight problem (algebra):
   - factor $16y^2 - 40yz + 25z^2$

2) Insight problem (nonroutine):
   • A prisoner was attempting escape from a tower. He found in his cell a rope which was half long enough to permit him to reach the ground safely. He divided the rope in half and tied the two parts together and escaped. How could he have done this?
Results (1)

• First result: subjects “feelings of knowing” (beforehand) only predicted eventual success of solving the problem for noninsight problems.

• At 15 seconds intervals, ss. rated how close they felt to solving the problem:

  1=cold (nowhere close to solution)

  ....

  7=hot (problem is virtually solved)
Number of times a particular warmth rating was given
Expertise
Developing Expertise

• What are differences between novices and experts?

• How to become an expert?
• Experts need only a few seconds to see what is wrong (or what isn’t)
• Experts perceive large meaningful patterns in their domain
Chess Studies

• De Groot (1965)

• Instructed 5 chess grandmasters to think out loud

• Grandmasters only considered about 30 moves and only thought 6 moves ahead.

• Not that different from novices. However, The 30 moves considered by a grandmaster are *really good* moves

• Masters rely on extensive experience: 50,000 patterns
Chase & Simon (1973)

Chess masters only expert with real chess positions. They do not have better memory in general.
Visual Expertise in Sports

• Some aspects of sports require extensive training and don’t transfer even to similar sports
• Example: visual skills in baseball and softball

https://www.youtube.com/watch?v=gm9iZnqGMvY
What makes an expert an expert?

• Talent? IQ? Practice? Genetic factors?

• Experts are masters mostly in their own domain; the skill does not cross into different domains

(Voss et al., 1983)

• Study exceptional feats:
  – Memory experts
  – Chess experts
  – Musicians
  – Athletes
10 year “rule”

• 10 years of **deliberate practice** needed to attain an international level

• **Deliberate practice**: practice that is highly motivated and involves careful self-monitoring

• Master chess players spend 10,000 – 20,000 hours playing

(Ericsson et al, 1993)
Difference between good and exceptional musicians is related to the amount of practice

Graph from Ericsson et al. (1996) showing the cumulative amount of practice by two groups of aspiring musical performers (experts and good violinists) and those who planned to teach music
Rule or Myth?

• 10 year ‘rule’ has been questioned by researchers

• Genetic factors? Maybe exceptional performance in some area can be explained by talent – an innate predisposition that predetermines performance in a domain (read “sports gene”)

• Importance of enjoyment and intrinsic motivation

• Age at which person starts to practice