Leveraging Response Consistency within Individuals to Improve Group Accuracy for Rank-Ordering Problems

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Abstract

Averaging the estimates of a number of individuals has been shown to produce an estimate that is generally more accurate than those of the individuals themselves. Similarly, averaging responses from a single individual can also lead to a more accurate answer. How can we best combine estimates within and between individuals to create an accurate group estimate? We report empirical results from a general knowledge rank-ordering experiment and demonstrate that individuals that provide more consistent answers across repeated elicitations are also more accurate. We develop a consistency weighting heuristic and show that repeated elicitations within an individual can be used to improve group accuracy. We also develop a Thurstonian cognitive model which assumes a direct link between the process that explains the accuracy of an individual and response consistency and show how the model can infer accurate group answers.

Keywords: Bayesian Modeling; Rank Ordering; Knowledge; Recall; Wisdom of Crowds; Within; Expertise; Uncertainty; Coherence; Consistency.

Introduction

There has been a lot of interest recently regarding how the judgments of individuals can best be combined to make group estimates that are as accurate as possible. When there is a ground truth – one single, verifiable correct answer – the group average is often more accurate than most or all of its constituent individual judgments (Davis-Stober, Budescu, & Broomell, 2014; Wallsten, Budescu, Erev, & Diederich, 1997; Yaniv & Foster, 1997) even if the correct answer is unknowable at the time of questioning (Lee, Steyvers, de Young, & Miller, 2012). When repeated judgments are averaged within one individual as opposed to across individuals, a similar phenomenon occurs. For example, when a single person produces two estimates for the same underlying quantity, the average of the two estimates is generally less erroneous than the individual estimates (Vul & Pashler, 2008; Herzog & Hertwig, 2009; Ariely et al. 2000). A standard explanation for these averaging benefits is that random error associated with probabilistic mental representations and processes partially cancel out in the average. A larger averaging benefit is typically found when averaging judgments across as opposed to within subjects (Rauhaut & Lorenz, 2011; Müller-Trede, 2011) presumably because differences in mental representations and associated random error is larger across individuals.

In order to improve the accuracy of the group average, many approaches have been developed to identify and upweight more expert or accurate judgments in the group average, including performance or contributor weighting (Budescu and Chen; Cooke, 1991; Bedford & Cooke, 2001; Aspinal, 2010), consensus (Shanteau et al. 2002; Wang et al. 2011; Batchelder & Romney, 1988; Batchelder & Anders, 2012; Lee, Steyvers, de Young & Miller, 2012; Lee, Steyvers, & Miller, 2014) as well as subjective confidence and metacognitive judgments (Koriat, 2012; Prelec, 2004).

We will focus on the role of response agreement within subjects as an indicator for expert judgment. Previous research has shown that expert judgments tend to be more consistent over time (Einhorn, 1972, 1974) and that intra-subject reliability can be used as a proxy for expertise (Shanteau, Weiss, Thomas, & Pounds, 2002; Weiss & Shanteau, 2003; Weiss, Brennan, Thomas, Kirlik, & Miller, 2009). This work has focused on the idea of highly specialized expertise and across-question consistency for tasks such as perception and categorization (Weiss & Shanteau, 2003; Weiss, Brennan, Thomas, Kirlik, & Miller, 2009). As opposed to previous research, we focus on tasks where expertise may be question-specific; subjects may have knowledge for some questions, but not for others, making their question level consistency more informative about their expertise than the overall domain consistency.

One challenge for using intra-subject consistency as an indicator for expert judgment is that other factors can contribute to response agreement, including decision strategies and episodic recall (Vul & Pashler, 2008; Hourihan & Benjamin, 2010). For example, in Vul and Pashler's experiment, subjects were prompted for a second estimate either in the same experimental session or after a delay of three weeks. The intra-subject averages were most accurate after a delay of three weeks, suggesting that subjects were less likely to simply recall the first answer after a long delay. The requirement of a long temporal delay between repeated questions to avoid episodic recall might not be practical in scenarios where subject judgments need to be aggregated over a short interval.

In this paper, we focus on rank-ordering questions where the task is to rank-order a set items such as Presidents by terms in office or US cities by population size (Miller, Hemmer, Steyvers, Lee, 2009; Lee et al. 2012; Lee, Steyvers, Miller, 2014). In contrast to simple yes/no or percentage estimation question involving single quantities, rank-ordering questions involve the retrieval and coordination of many pieces of information, making it less likely that a subject can explicitly remember a previous response. In the absence of easily available episodic strategies, subjects can be asked for a second response almost immediately after their first, eliminating the need for multiple conditions and removing any question anchoring effects.

Our contribution in this paper is threefold. First, we show that the crowd within an individual effect observed by Vul and Pashler exists for rank-ordering tasks, indicating that there is a degree of statistical independence between repeated elicitations for rank-ordering judgments. Second, we demonstrate that the agreement between the first and second response is related to each subjects' response accuracy. We present a simple consistency weighting heuristic where rank-ordering judgments from individuals that are consistent across repeated questions are given larger weight in the group average. We demonstrate that this consistency weighting heuristic significantly improves group accuracy. Finally, we introduce a new repeated-elicitation variant of a Thurstonian model for rank-ordering that has been explored elsewhere (see Steyvers et al., 2009 & Lee et al., 2012). We compare the performance of the repeated-elicitation model and the original variant, and demonstrate that accounting for the variance in an individual's responses improves overall group aggregation performance

Experiment

Method

The experiment was composed of 8 rank ordering questions, and an additional 3 distracter questions; the distracter questions were included to increase the delay between subject responses. Increased delay between responses has been shown previously to increase response independence and effect size (see Vul & Pashler, 2008). Subjects were 120 undergraduate students between the ages of 18 and 22 at the University of California, Irvine who were compensated with course credit.

Selection for the non-distracter questions was based on difficulty, as determined by the accuracy of subjects in previous experiments (Steyvers et al., 2009; Miller et al., 2011). Approximately one third of questions were selected for being easier (U.S. Holidays, U.S. Presidents, Book Release Dates), three for being moderately difficult (Country Landmasses, U.S. Cities, European Cities), and one two for being particularly difficult (10 Amendments, World Cities). All were general knowledge questions that subjects were likely to have had exposure to. For the distracter questions, subjects were asked to rank teams for the NFL and NBA based on what they thought their final season standing would be.

Subjects were given the eight knowledge questions in a random order, and items for each question were initially placed in random positions. Subjects were then given the distracter questions. Subjects were then prompted to give responses for the eight questions again, in the same order they appeared in the first elicitation, but with a new random initial placement of the items for each question.

All questions had a ground truth obtained from Pocket World in Figures and various online sources. An interactive interface was presented via a web browser on computer screens. Subjects were instructed to order the presented items (e.g., "Order these books by their first release date, earliest to most recent"), and responded by dragging the individual items on the screen using the computer mouse and "snapping" them into the desired locations in the ordering, as in previous experiments. Transitions between question blocks were marked by a holding page reminding subjects of the instructions for the tasks. At no point were subjects informed that they would be answering the same questions twice.

Results

Assessing Accuracy Performance was measured relative to the ground truth using Kendall's tau distance τ. This metric is used to count the number of pair-wise disagreements between the reconstructed and correct ordering (lower is better). The larger the distance, the more dissimilar the two orderings are. Values of τ range from: $0 \leq \tau \leq N(N-1)/2$, where N is the number of items in the order (ten for all of our questions). A value of zero means the ordering is exactly right, a value of one means that the ordering is correct except for two neighboring items being transposed, and so on up to the maximum possible value of forty-five (indicating that the list is completely reversed). An average score of 22.5 is expected for random performance.

Averaged Responses We first evaluated whether or not averaging the responses within each individual reduced the error relative to the individual responses, indicating statistically independent error of the sort observed in the simple recall tasks of Vul and Pashler (2008). Table 1 shows the median Kendall's tau distance for individual rank-ordering problems for the first and second response as well as the combined first and second response using the Borda aggregation method (see modeling section for Borda details). Subjects' error on the first and second responses were not significantly different, on average, and varied according to question difficulty. The averaged first and second responses of each subject (combined column in Table 1) was less erroneous than the first and second responses – t(120)=2.16, p<.05 and t(120)=2.87, p<.01 respectively – replicating the findings of Vul and Pashler (2008) for rank ordering tasks.

Table 1: *Subject performance error (Kendall's tau) across individual rank-ordering problems.*

Problems	1^{st}	2^{nd}	Combined
Landmass	9	10	8
Holidays	7	8	7
Presidents	7	7	6
Books	11	11	10
Euro Cities	15	16	14
US Cities	16	14	14
World Cities	21	21	20
10 Amendments	16	15	15
AVERAGE	12.5	12.7	11.9

Response Consistency and Accuracy If subject response consistency is correlated to the precision of an individual's knowledge, then multiple independent responses should be further apart from each other the less knowledgeable a subject is. We quantified (inverse) response consistency as the Kendall's tau distance between subjects' responses. Subjects with a larger distance between their first and second judgment should show a higher tau distance to the ground truth. Figure 1 illustrates this relationship separately for the first and second response. The correlation between each subject's response disagreement, and the error of their first and second responses, is ρ=.51 and ρ=.55 respectively. This correlation is observed not only across all questions, but also for each individual question. The correlation between response disagreement and accuracy appears to scale linearly with overall subject accuracy for the problem.

Modeling

While averaging across a given individual's responses yields answers that are more accurate, the improvement is far smaller than averaging two responses *across* subjects (Miller et al., 2011). Given a large number of subjects, it is unclear whether repeated elicitations would improve group responses aggregation if they are merely treated as extra subjects. Can within-subject response consistency be integrated into a between-subject aggregation model to improve overall accuracy? To test this, we evaluate two models – a heuristic approach based on Borda aggregation method and a Thurstonian cognitive model of subject behavior.

Borda Aggregation

In order to assess if incorporating within-subject response consistency can improve between-subject estimates for rank ordering tasks, we used a modified version of Borda count aggregation that incorporates subject weighting. Borda aggregation is a representative aggregation heuristic that has been used widely elsewhere (see Miller et al., 2009). In traditional Borda count aggregation, all items are assigned points based upon their location in a given response: 1 point for being in position 1, 2 points for being in position 2, up to 10 points for a list of 10 items. In a standard Borda aggregation method, the points are added across all rank-orderings provided by subjects and the items are ordered according to the sum totals for each item. In our modified Borda aggregation method, we add a weighting factor for each individual subject in order to upweight subjects that are more consistent. Specifically, we calculate the point total $s_k$for each item $k \in \{1, \dots, K\}$ by:

$$s_k = \sum_{j=1}^{N} r_{j,k} w_j$$

where $r_{j,k}$ is the rank of item k for subject $j \in \{1, \dots, N\}$, and w_j is the weight given to subject j. As in a standard Borda method, the sums of these points for each item are then ranked from smallest to largest to determine the final Borda aggregate rank ordering

For an unweighted aggregate rank-ordering, the subject weights were set to the same value for all participants. We used this as the baseline for comparison. For the aggregate rank-ordering weighted by response consistency, we use the inverse of the tau disagreement between the first and second rank-ordering:

$$w_j = 1/(\tau_j + 1)$$

where we add one to the distance in order to avoid zero division. Therefore, the rank-orderings of participants with larger response consistency have a stronger influence on the aggregate rank ordering.

Figure 2 shows the aggregation results. As we found previously (Miller et al., 2009), unweighted Borda

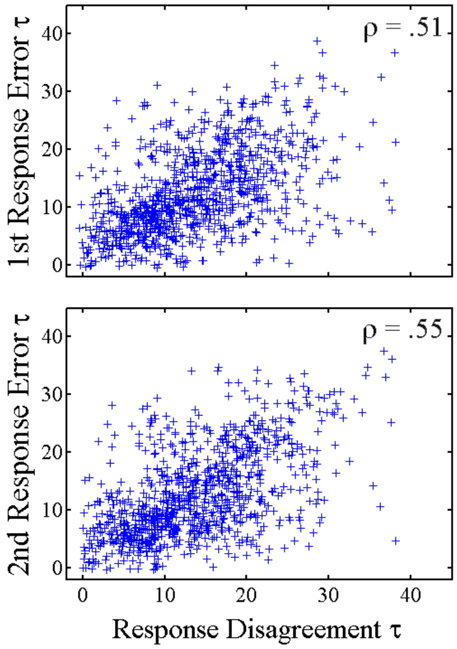


Figure 1: *Correlation between response disagreement and accuracy for the first answer (top panel) and second answer (lower panel).*

aggregation outperforms the average subject for all eight questions. Additionally, the weighted Borda model performs as well as or better than the unweighted model for all but two of the questions. The weighted Borda model performed worse for the Presidents question because most subjects performed so well that weighting over-penalized the many subjects with near-correct responses. Similarly, the model performed poorly on the European Cities question because there were so few subjects that performed well. Aggregation for the unweighted Borda model was performed across both trials so as not to give the weighted model the advantage of extra subject responses. This superior performance in reconstructing the ground truth ordering demonstrates that response consistency can be used to improve group accuracy for rank ordering tasks. Next we explore whether a cognitive model of the rank-ordering task can better describe subject behavior and more accurately reconstruct the ground truth.

Thurstonian Model

Given that subject response consistency is clearly related to accuracy in rank-ordering tasks, what kind of mechanism might be responsible for this observed behavior? We developed a probabilistic model based upon a Thurstonian approach. In a Thurstonian representation, the latent ground truth ordering for a specific problem is represented by coordinates on an interval scale. As Figure 3a illustrates, each item *k* is represented as a latent coordinate μ_k on an interval dimension. Note that this represents not the actual ground truth but the latent truth as perceived by a group of individuals. The one-dimensional representation of items is appropriate as all problems in our study involve one-dimensional relative judgments (e.g. the size of items and the timing of events).

Each individual *i* is assumed to have access to all of the ground truth latent coordinates $\boldsymbol{\mu}$, but without precise knowledge about their exact locations. This uncertainty is represented with normal distributions that are centered on the shared latent ground truth locations and with a subject-level σ_i that represents the uncertainty of the individual about the item locations. Note that for a given subject, all items have the same standard deviation which is a strong assumption but simplifies the model considerably.

As Figure 3b shows, the subject draws mental samples from these item distributions. Repeated elicitations are modeled simply by repeating the sampling process which leads to a new set of samples. The rank-ordering produced by a subject is then based on the order of the mental samples.

As illustrated in Figure 3c, different subjects can have different uncertainty σ_i, and this influences not only the response accuracy but also the response consistency. For example, the larger uncertainty associated with the subject illustrated in Figure 3c leads to more transposition errors in the mental samples associated with a given response – it becomes more likely that samples of nearby distributions are out of order (relative to the latent ground truth) which lowers accuracy. In addition, the larger uncertainty also leads to increased differences in orderings between different responses. Therefore, the model assumes an inherent connection between response consistency and accuracy – they are both driven by a latent parameter σ_i that represents the (inverse) expertise level of a subject for a particular

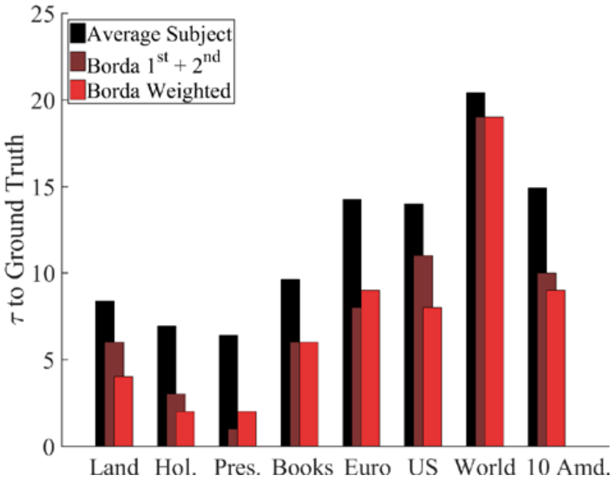
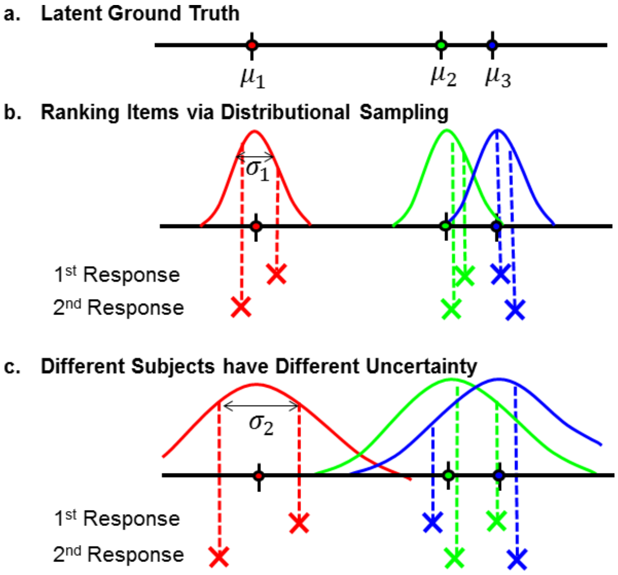


Figure 2: Aggregation performance of unweighted and weighted Borda aggregation across first and second responses, compared to the average subject performance.



a. Latent Ground Truth
μ1
μ2
μ3
b. Ranking Items via Distributional Sampling
σ1
1st Response
2nd Response
c. Different Subjects have Different Uncertainty
σ2
1st Response
2nd Response

Figure 3: Illustration of the Thurstonian Model for repeated elicitations. (a) The latent ground truth is represented as a set of coordinates on an interval scale (b) Uncertainty about the latent ground truth is represented by Gaussian noise and responses are created by sampling latent values from each item distribution (c) Example of a subject with larger uncertainty about the ground truth and larger variability in the item samples across the first and second response

question.

This multiple-elicitation model is different from previous Thurstonian models that we have presented, where subjects only give a single response per question (Steyvers et al., 2009; Miller & Steyvers, 2011). This extended model allows us to examine whether accuracy and response consistency can be described with the same underlying mechanism.

We apply Bayesian estimation techniques to infer the group representation from individual orderings. Figure 4 shows the Thurstonian model for a single question across subjects using graphical model notation (see Koller, Friedman, Getoor, & Taskar, 2007; Shiffrin, Lee, Kim, & Wagenmakers, 2008, for statistical and psychological introductions). Each node represents a model variable, and the graph structure is used to indicate the conditional dependencies between these variables. Stochastic and deterministic variables are indicated by single-and double-bordered nodes ($\boldsymbol{\mu}$, σ, x and y respectively), and observed data are represented by a shaded node (y). The plate represents independent replications of the graph structure, which corresponds to multiple elicitations from each individual i and across individuals for each question j.

To explain how these data are generated, the model begins with the underlying ground truth location of the items, given by the vector $\boldsymbol{\mu}$. The latent ground truth μ is given a flat prior such that all item locations are equally likely a priori. Each individual has an associated uncertainty parameter σ_j ~ Gamma($\lambda\sigma_0$, $1/\lambda$) where λ is a hyper-parameter that determines the variability of the expertise levels across individuals. We set $\lambda = 3$ in the current model.

To determine the order of items for the ith repetition, the jth individual samples a location x_{ijk} for each item k where x_{ijk} ~ Normal(μ_k,σ_j). The sample x_{ijk} represents the realized mental representation for the individual at that particular time. The ordering for each individual is determined by the ordering of all of their mental samples y_{ij} = Rank($\boldsymbol{x}_{ij}$).

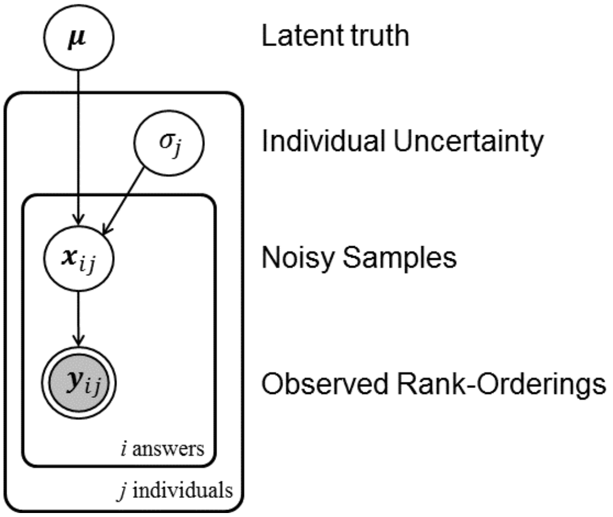


Figure 4: Graphical model of the Thurstonian model for repeated elicitations.

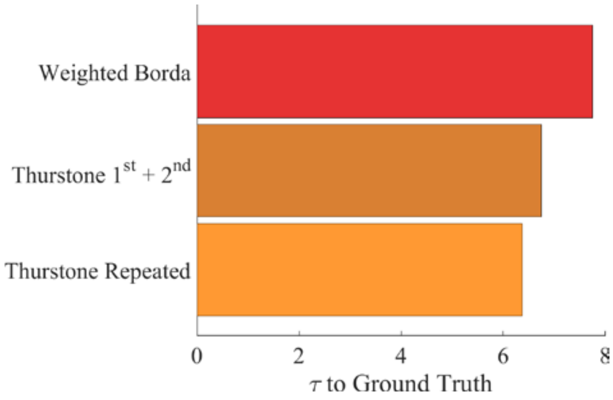


Figure 5: Aggregation performance of weighted Borda, traditional Thurstonian, and repeated Thurstonian models.

While the generative model is relatively straightforward, the inference is challenging because the observed data y_{ij} is a deterministic ranking. We utilized MCMC procedures originally developed by Yao and Böckenholt (1999), which allowed us to estimate the posterior distribution over the latent variables x_{ijk}, σ_j, and $\boldsymbol{\mu}$ given the observed orderings y_{ij}. We use Gibbs sampling to update the mental samples x_{ijk}, and Metropolis-Hastings updates for σ_j and $\boldsymbol{\mu}$.

Figure 5 shows the accuracy of three aggregation models, and demonstrates that the repeated elicitation Thurstonian model performed best overall. It outperformed the weighted Borda model and also outperformed a Thurstonian model that is given *both* the first and second response of participants but treats the second responses as coming from a new set of participants. Additionally, the repeated elicitation Thurstonian model matched or exceeded other models' performance for each individual question.

The advantage of the repeated elicitation Thurstonian model over the Thurstonian model where the first and second responses are not linked to the same subject is not due to the fact that it has access to additional response information (it uses the same set of subject responses), but because the model simultaneously infers a subject's uncertainty based upon their disagreement with other subjects and their disagreement with themselves. In this way, we have some confidence in the Thurstonian representation of individual-level uncertainty for subject item recall, both as a generative model and as a means of yielding more accurate group estimates for rank ordering tasks.

Conclusions

In this paper, we have shown that repeated elicitations for general knowledge rank-ordering tasks exhibit statistically independent error, and the variance of that error is correlated to the accuracy of subject responses for easy and difficult questions. Additionally, we have shown that this response consistency can be used to improve group aggregate accuracy in reconstructing the ground truth answer for rank ordering knowledge tasks. These findings might also be applicable to tasks that do not have a known ground truth, as we have discussed elsewhere (Lee et al., 2012). Finally, we introduced a cognitive model of rank-order judgement wherein a subject-level uncertainty parameter accounted for both subject response accuracy and response consistency, and found that it was best able to capture subject behavior and reconstruct the original ground truth ordering for each of our questions. This lends credence to the idea of a combined probabilistic mechanism for consistency and accuracy underlying the subject behavior observed in these complex knowledge recall tasks.

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