

A Bayesian model of knowledge and metacognitive control: Applications to *opt-in* tasks

Stephen T. Bennett (stbennet@uci.edu)

Department of Cognitive Science
University of California, Irvine

Aaron S. Benjamin (asbenjam@illinois.edu)

Department of Psychology and Beckman Institute for Advanced Science and Technology
University of Illinois at Urbana-Champaign

Mark Steyvers (mark.steyvers@uci.edu)

Department of Cognitive Science
University of California, Irvine

Abstract

In many ecologically situated cognitive tasks, participants engage in self-selection of the particular stimuli they choose to evaluate or test themselves on. This contrasts with a traditional experimental approach in which an experimenter has complete control over the participant's experience. Considering these two situations jointly provides an opportunity to understand why participants opt in to some stimuli or tasks but not to others. We present here a Bayesian model of cognitive and metacognitive processes that uses latent contextual knowledge to model how learners use knowledge to make opt-in decisions. We leverage the model to describe how performance on self-selected stimuli relates to performance on true experimental tasks that deny learners the opportunity for self-selection. We illustrate the utility of the approach with an application to a general-knowledge answering task.

Keywords: metacognitive control; Bayesian cognitive model; wisdom of the crowd; opt-in; missing not at random

Background

In traditional approaches to experimental psychology, an experimenter has unilateral control over which stimuli a participant experiences and the tasks that they complete. Yet in many real-world situations, such as providing ratings to videos on the Internet, the participant has some or even total control over the specific stimuli and tasks that they experience. The choice behavior underlying such self-selection is an important domain of study called *metacognition* (Nelson & Narens, 1990), and the self-selection of activities or stimuli is specifically called *metacognitive control* (Fiechter, Benjamin, & Unsworth, 2016; Finley, Tullis, & Benjamin, 2010). Some work on monitoring and control processes in memory tasks focused on confidence judgments as an indicator of self-selection questions (Kelley & Sahakyan, 2003; Koriat & Goldsmith, 1996). It is unclear precisely how this self-selection is generated, however. To better understand metacognitive control behavior, a model is needed that accounts for performance on the task of interest as well as the choice behavior that leads participants to select only some stimuli for exposure, evaluation, or testing.

The major difficulty of such an endeavor is that participants select tasks according to their interests and expertise, and so

the data is missing in a nonrandom fashion (see Little & Rubin, 2014, for a description of other missing data scenarios). Consequently, participants can only be compared and their performance fairly evaluated if a model is specified for the opt-in process. If a participant does not opt in to a particular question, then we simply do not see that participant's response to that question.

A starting point in explaining opt-in behavior is that participants have some meta-knowledge of what it is they already know, and use that knowledge effectively in service of ongoing learning. People provide higher assessments of their ability to answer inference questions in domains in which they have greater expertise (Bradley, 1981), and learners often choose to engage more effective study techniques for material that is more difficult for them (A. S. Benjamin & Bird, 2006). Memory reports are also considerably more accurate when respondents have the option of withholding answers that they are unsure of or of titrating the grain size of their answers to their perceived accuracy (Goldsmith & Koriat, 2007).

Self-regulated learning often has substantial benefits in educational contexts (Mezirow, 1981; Zimmerman, 1989; Boekaerts & Minnaert, 1999; Paris & Paris, 2001). Learners use meta-knowledge to allocate time, resources, and activities to an array of learning goals, and this application increases overall performance compared to learners who have their learning activities dictated by an instructor (Winne & Hadwin, 1998; Finley et al., 2010).

The benefits of self-control extend beyond these constrained tasks, however. In causal reasoning experiments, participants can more quickly understand the causal structure of a network if they intervene in the learning process and design their own "experiments" (Steyvers, Tenenbaum, Wagenmakers, & Blum, 2003; Sobel & Kushnir, 2006; Lagnado & Sloman, 2004). Human strategy selection can be explained in terms of rational metareasoning, wherein humans flexibly choose strategies in accordance with their environment (Lieder & Griffiths, 2015; Lieder et al., 2014).

The core claim across each of these examples is that self-selection within a task aimed at measuring performance is

driven by metacognitive knowledge, which leads to a higher rate of success, expertise, or interest for the selected items. This process makes it difficult to evaluate the stimuli and the participants in an unbiased way. One test-taker may, for example, outperform another not because they have greater knowledge but rather because they make more a judicious selection of problems.

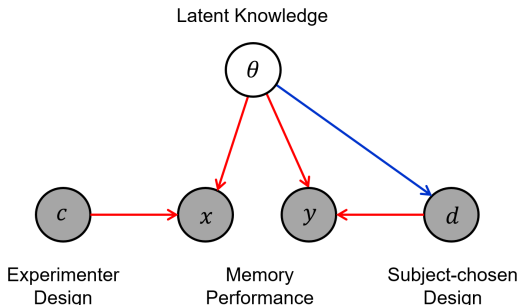


Figure 1: Outline of our modeling approach. Latent knowledge and design both explain the performance on the task. In the case of a subject-chosen design, latent knowledge also explains the design.

The aim of this project is to develop a cognitive model of the metacognitive aspect of item selection. In doing so, it also provides a framework to relate performance on self-selected materials with performance on an unconstrained set of items or stimuli. Here we apply this model to data collected from participants answering general knowledge questions, but the model is considerably more general: the same principles could apply in other metacognitive control tasks, such as study time allocation or selection of items for restudy. We are aware of one current model of metacognitive control, which takes as a given the state of the world, which then causes the observed behaviors (Fleming & Daw, 2017). We take a different approach, which starts with the latent knowledge that the participant is coming to the experiment with and uses that in both the selection process behind opting-in and the observed responses to questions, as illustrated in Figure 1. Performance on the task is explained by both the design—that is, the particular experience of the participant in the task—and the latent knowledge of the participant. In the case of a participant who can opt in to certain questions but avoid others, the design is also partially informed by the latent knowledge. We are interested in estimating the latent knowledge of each participant and evaluating how it relates both to performance on the task and to opt-in behavior. In order to infer latent knowledge from the observed data, we apply Bayes’ rule in the equation below, where θ is the latent knowledge, c is the experimenter design, d is the subject-chosen design, x are the performance data from a true experimental design (where subjects respond to all or to a random subject of probes), and y are the performance data from a subject-chosen design (in which subjects choose which probes to re-

spond to):

$$p(\theta|c, d, x, y) \propto p(x|\theta, c)p(y|\theta, d)p(d|\theta)p(\theta) \quad (1)$$

In a traditional cognitive model, the important part of the model is the specification of $p(x|\theta, c)$ and $p(y|\theta, d)$, termed the likelihood functions. These functions directly explain the empirical effect of interest by relating latent knowledge to performance on the task given the experimental design. The novel part of the model relates to the specification of the metacognitive control process $p(d|\theta)$, which explains how the participant self-designs on the basis of their latent knowledge. If we would ignore this model component, we would likely, and incorrectly, conclude that participants who self-designed were more knowledgeable than participants subject to the experimenter’s design because they outperformed their experimenter-designed counterparts. Such an error could be catastrophic if we were trying to compare across individuals or across tests. Because subjects are randomly assigned to conditions, it is highly unlikely that they differ widely. The process by which the participants who self-designed outperformed those who could not lies in the opportunity to self-design. Here we see the importance of jointly modeling the selection process and the task at hand in order to understand the interplay between latent knowledge, opt-in behavior, and performance.

Since this is a task in which many participants give judgments to many questions, we also expect to find that averaging across participants leads to higher accuracy—an effect termed the *wisdom of the crowd* (Surowiecki, 2004; Steyvers, Miller, Hemmer, & Lee, 2009). Here we have the opportunity to evaluate whether the opportunity to opt in to a self-selected portion of the questions will enhance or attenuate such benefits associated with averaging. Certainly, many participants will gravitate towards the same questions when they can opt in, which would potentially decrease the benefits of averaging across a crowd by virtue of reducing input to the more difficult questions. However, based on what is known about metacognition, we expect that participants will opt in to questions for which they have relevant knowledge, which could lead to a more informed set of responses to average with the remaining crowd. Crowd behavior provides an additional benchmark against which we can evaluate the performance of the metacognitive model.

Experiment

Stimuli The question set consisted of 100 general-knowledge binary choice questions. The questions were drawn from 12 topics: World Facts, World History, Sports, Earth Sciences, Physical Sciences, Life Sciences, Psychology, Space & Universe, Math & Logic, Climate Change, Physical Geography, and Vocabulary. The question set was created by collecting from multiple sources. Two example questions are shown in Table 1. Based on the empirically observed accuracy levels, the first is difficult and the second is easy.

Participants A total of 83 participants were recruited through Amazon Mechanical Turk (AMT). Each participant

Table 1: Example questions.

Difficulty	Example
Hard	The Sun and the planets in our Solar system all rotate in the same direction because: (a) they were all formed from the same spinning nebular cloud, or (b) of the way the gravitational forces of the Sun and the planets interact
Easy	Greenhouse effect refers to: (a) gases in the atmosphere that trap heat, or (b) impact to the Earth’s ozone layer

was compensated \$1 for the 30 minutes the experiment was expected to take, and assigned to one condition.

Design Participants could view the survey description on AMT. If they selected the survey they were redirected to another website. They were first directed to a study information sheet which provided details of the survey and compensation. If they agreed to continue, they were instructed to answer some demographic questions. Participants were randomly assigned to either a *random* condition ($N = 44$) or a *self-selection* condition ($N = 39$), determining the subject’s role in selecting which questions to answer. Participants were not aware of the existence of other conditions. Each participant saw the questions in 5 blocks of 20 questions each. In each block, they were instructed to rate the difficulty of each question and then, if they were assigned to the opt-in condition, instructed to choose 5 of those 20 questions to answer. The participants in the random assignment condition were randomly assigned 5 questions from that block to answer. After rating the difficulty of all 100 questions and answering 25 of them, participants were thanked for their time and given instructions on how to receive payment.

Model

The model utilizes an IRT model to generate subjective latent knowledge (the belief of a participant that she can answer a question), which informs all aspects of participants’ responses including the observed accuracy and difficulty ratings, as well as the metacognitive process of question selection. We describe participants as opting-in to questions for which they believe they have knowledge, answering with accuracy dependent on whether or not they believe they have knowledge, and giving lower difficulty ratings when they believe they have knowledge.

We use an IRT model to generate the subjective latent knowledge, $\delta_{i,j}$, for each participant i (across both the opt-in and random condition) and question j ,

$$\delta_{i,j} \sim \text{Bernoulli}(\text{logit}^{-1}(\theta_i + \eta_j)) \quad (2)$$

where θ_i is the self-perceived skill of participant i , η_j is the perceived familiarity of question j , and $\text{logit}^{-1}(x) = \frac{e^x}{1+e^x}$. This latent knowledge is represented as a 0 or 1, indicating whether or not that participant believes that she has knowledge for that question. We place a Normal prior on the self-perceived skill, $\theta_i \sim \text{Normal}(0, \sigma)$, such that participants are expected to have the same skill (on average) for both the self-selection and random conditions.

For the self-selection condition, we assume that participants have a preference to select questions for which they believe they have knowledge. Let c represent the observed question selections with $c_{i,j} = 1$ if question j was selected by participant i . For each participant and question block, we model question selection in the opt-in condition by a sampling process:

$$c_i \sim \text{SampleWR}((\delta_{i,1} + \kappa, \dots, \delta_{i,K} + \kappa), M) \quad (3)$$

where K is the total number of questions available for selection in each block ($K=20$ in our experiment), M is the number of questions that need to be selected ($M=5$ in the experiment), $\text{SampleWR}(\delta, M)$ represents a sampling without replacement distribution where M items are sampled with probability proportional to δ , and κ is a fixed parameter that controls the randomness in the selection process. Higher κ values make it more likely that questions are selected for which the participant has no subjective knowledge. For participants in the random condition, we assume that the questions are randomly sampled by a process that is under control of the experimenter (where M out of K questions are randomly allocated).

Let $x_{i,j}$ represent the observed accuracy for participant i on question j . We do not assume a fixed relationship between belief of knowledge and accuracy. For each question, we introduce guessing rate parameters ρ_j and λ_j that control the probability of correct responding if the participant does or does not have subjective knowledge about a question:

$$x_{i,j} \sim \text{Bernoulli}(\delta_{i,j}\rho_j + (1 - \delta_{i,j})\lambda_j) \quad (4)$$

For example, with $\rho = 0.8$ and $\lambda = 0.4$, the probability of a correct response is 0.8 if a participant has subjective knowledge, but 0.4 if the participant does not. The guessing parameters are given Beta priors, $\rho_j \sim \text{Beta}(\alpha, \beta)$, $\lambda_j \sim \text{Beta}(\alpha, \beta)$ where α and β are hyperparameters that control the variability in guessing rates across questions.

To model the difficulty ratings, we use an ordered logit model (Williams et al., 2006). We assume that subjective latent knowledge informs the perceived difficulty of questions. Questions for which the participant believes they have knowledge are perceived as easier. Let $\phi_{i,j}$ represent the perceived difficulty for participant i on question j . We determine the perceived difficulty by:

$$\phi_{i,j} = -\delta_{i,j} - \eta_j \xi + \omega_i - \beta_j + \sigma_{i,j} \quad (5)$$

where β_j and ω_i capture participant and item level effects (e.g. some participants might find all items easy, some items

might be judged as easy) independent of subjective knowledge. In addition, we also allow the perceived familiarity of a question η_j to affect the perceived difficulty weighted by a fixed scaling parameter ξ . Finally, $\sigma_{i,j}$ represent small perturbations centered around 0 to explain the random variability in difficulty ratings unrelated to any of the previous factors mentioned. These perceived difficulties feed into the ordered logit model to generate the difficulty ratings $r_{i,j}$,

$$r_{i,j} \sim \text{OrderedLogit}(\phi_{i,j}, \tau_i) \quad (6)$$

where τ_i is the set of criteria cutoffs for participant i .

We used JAGS to perform parameter inference. All parameters were inferred jointly from the opt-in and random condition. All model predictions were derived from posterior predictives where we simulate new participants from the distribution and assess how they self-select from a new set of questions.

Results

We examine several empirical effects within the data and observe that the model captures the appropriate trend in most cases.

Item selection and latent knowledge. The model captures the expected relationship between opting-in behavior and knowledge (see Figure 2). Participants were more likely to select questions for which they had pre-existing knowledge. Each question was randomly assigned to at least four participants in the random assignment condition. However, in the opt-in condition, there were seven questions that no participant chose to answer. Question selection strongly corresponded with the inferred latent knowledge ($\delta_{i,j}$) for the participant-question pair, with participants choosing questions for which they had latent knowledge. Across conditions, latent knowledge is distributed in a similar manner: most participants have knowledge for popular questions, few participants have knowledge for unpopular questions, and some participants are more knowledgeable than others. However, the model has substantially more certainty about the localization of this knowledge in the opt-in condition compared to the random condition because it can leverage the opt-in behavior. In Figure 2, this certainty is expressed as black or white squares, while uncertainty is represented in gray. We see the uncertainty about which participants have knowledge for which question as a “blurring” of the latent knowledge space.

Effect of opting-in on participant performance. Average performance across questions was higher in the self-selection condition (86.05%) than in the random condition (67.27%). We computed a Bayes Factor (BF) given a binomial distribution with a shared or different rate of correct responding and find a Log_{10} BF of 21.12 in favor of a higher rate of correct responding in the opt-in condition. This corresponds to decisive evidence that average accuracy is higher in the opt-in condition than the random assignment condition. This occurs even when taking into account the fact that people tend to opt

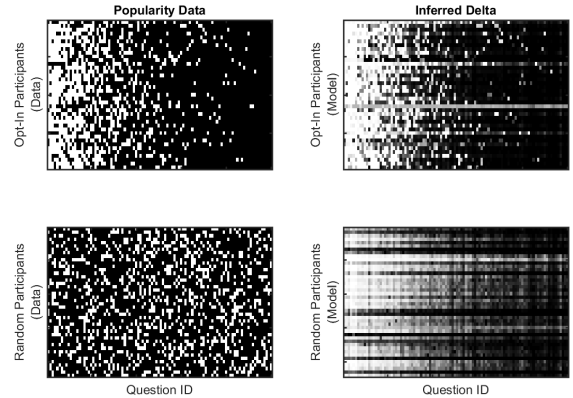


Figure 2: Latent knowledge is similar between conditions and corresponds to opting-in behavior. Plotted are the opt-in behaviors and average $\delta_{i,j}$ values across conditions, all sorted by the popularity of the question in the opt-in condition. White corresponds to questions that the participant opted in to or the inferred presence of knowledge.

in to easier questions. In order to perform this analysis, we took the product of the evidence that performance is higher in the opt-in condition than the random assignment condition for each question and find a Log_{10} BF of 9.02. So, even when comparing on an item-by-item basis, opting-in provides an advantage.

Effect of opt-in on model performance. For the model, the average accuracy for posterior predictive samples in the self-selection condition (mean = 79.03%) is also significantly higher than in the random condition (mean = 67.07%), both across all questions (99.86 % of samples) and even within questions (68.93 % of sample-question pairs). We observe this benefit in accuracy despite the average inferred ability of individual subjects (θ_i) being equivalent across conditions: $\bar{\theta}_i = 0.00$, $\text{SD} = 0.99$ in the opt-in condition versus $\bar{\theta}_i = -0.09$, $\text{SD} = 2.04$ in the random assignment condition. This means that the benefit to accuracy that the model predicts is due to downstream consequences of the metacognitive selection process and not an (inaccurate) inference that participants in one condition were more skillful than in the other.

Difficulty Ratings. Participants tended to give lower average difficulty ratings to questions that they opted in to (Log_{10} BF = 91.89) and higher average difficulty ratings to questions that they did not opt in to (Log_{10} BF = 64.09), relative to the random condition. The model captures, but understates, this trend (see Figure 3).

Wisdom of the crowd. The left panel of Figure 4 shows the relationship between crowd size and crowd accuracy for the two conditions in the experiment, as well as a hybrid condition in which the two groups are combined. The right side of the Figure shows that the model captures this effect qualitatively. Crowd responses were determined by taking the most

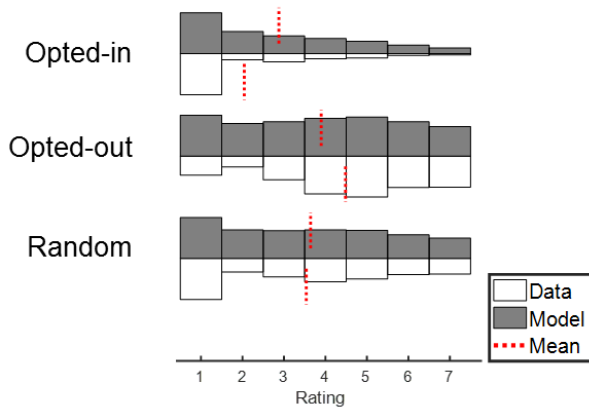


Figure 3: Distribution of difficulty ratings for participants and model for questions that were selected or not selected in the opt-in condition and the random condition. Lower ratings indicate lower perceived difficulty.

common response across the participants in the crowd. Since seven questions went unanswered in the opt-in condition, we had to consider how unanswered questions impacted crowd performance. To treat the self-selection condition maximally conservatively, we graded any question that went unanswered as incorrect for that crowd. Even with this penalty, the crowd composed of the participants from the self-selection condition (79%) outperformed the crowd of subjects from the random condition (73%).

We also considered the impact of crowd size on performance. To do this, we evaluated the average performance of crowds composed of random samples of participants from a condition and varied the number of participants drawn to form the sample. We plot average crowd performance as a function of the total number of judgments, where a judgment is a person’s response to a question. The hybrid condition provides a means of improving upon both conditions. To create a hybrid crowd, we first sampled participants that answered the question from the opt-in condition. If a question had no responses, we added the answer from one participant in the random condition in order to guarantee that all questions received at least one answer. This hybrid crowd has high performance across all questions. The model captures the general trends in the data in that larger crowds result in higher crowd accuracy, opt-in crowds outperform random-assignment crowds, and the hybrid crowds perform well across all questions.

Additional simulations. Given our model, we investigated which circumstances would likely lead to changes in the relative performance of the self-selection and random conditions in terms of both average overall accuracy and crowd performance. We varied the heterogeneity of perceived question difficulty (η_j) and latent ability (θ_i). We did this by simulating experiments in which we varied the underlying hyperparameter corresponding to the variability of θ_i and η_j by factors of 0.25, 1, and 4 while keeping other parameters constant

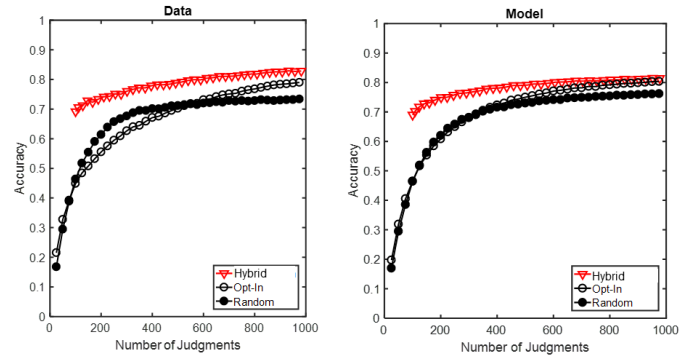


Figure 4: Crowd performance when varying the number of participants (measured by the total number of judgments)

(see Figure 5). We find that increasing the heterogeneity of perceived question difficulty increases self-selection accuracy overall, but decreases it at the crowd level since participants tend to avoid answering the same difficult questions. Heterogeneity in question difficulty does not have an appreciable impact on performance in the random condition. In both conditions, higher heterogeneity of participant skill leads to higher crowd performance and gives resilience to heterogeneously difficult questions in the opt-in condition. However, it detracts from overall accuracy in the self-selection condition.

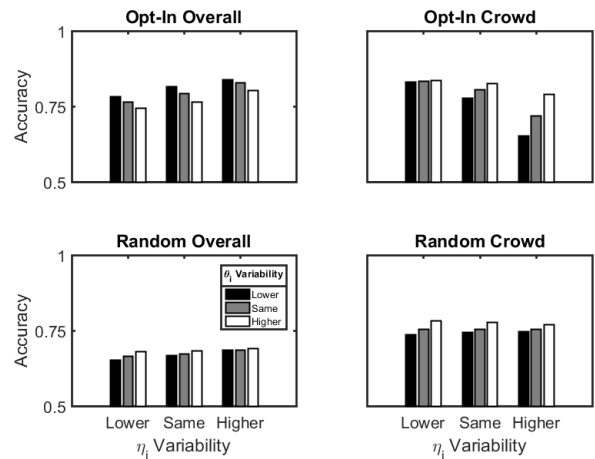


Figure 5: Simulated performance depending on variability of question difficulty (η_j) and participant skill (θ_i).

Conclusions

A comprehensive model of cognition must make allowance for the fact that cognitive behavior is driven by motivations. We choose what we attend to and attempt to encode, and what we attempt to remember. Metacognitive behavior is at the heart of most learning outside the laboratory, and a fair amount within it as well (A. Benjamin & Ross, 2008). The joint modeling of metacognitive behavior–like self-selection

of items—along with cognitive performance has the potential to address a wider and more representative range of real-world learning and testing behaviors, and can serve as the basis for drawing comparisons across individuals or tests that would otherwise be hopelessly confounded. Additionally, the model could be extended to explain various incentives given to the participant, which would impact how latent knowledge interacts with the task to generate opt-in behaviors. The model presented here provides a starting point for such an enterprise. It leads to a relatively good description of performance across a variety of metrics. A single latent knowledge state for each participant-question pair permits an explicit representation of the metacognitive process that governs the relationship between opt-in, accuracy, and difficulty behaviors. The model is successful in describing the nonrandom missing nature of the data that we observed by relying on principled psychological theories about why someone might choose one question over another.

An additional lesson of the current research can be seen in the crowd data. Opting in is generally beneficial to crowd accuracy in both the observed data and our model. This result indicates that the metacognitive skill of the individuals in self-selection can be leveraged in order to create a smarter crowd. This effect is sufficiently robust that it appears to outweigh the cost associated with small crowd sizes for some questions or no volunteered responses at all for a small number of questions. Such a result is particularly important when considering the widespread availability of datasets in which responses are self-selected.

References

- Benjamin, A., & Ross, B. H. (2008). *The psychology of learning and motivation: Skill and strategy in memory use* (Vol. 48). Academic Press.
- Benjamin, A. S., & Bird, R. D. (2006). Metacognitive control of the spacing of study repetitions. *Journal of Memory and Language, 55*(1), 126–137.
- Boekaerts, M., & Minnaert, A. (1999). Self-regulation with respect to informal learning. *International journal of educational research, 31*(6), 533–544.
- Bradley, J. V. (1981). Overconfidence in ignorant experts. *Bulletin of the Psychonomic Society, 17*(2), 82–84.
- Fiechter, J. L., Benjamin, A. S., & Unsworth, N. (2016). 16 the metacognitive foundations of effective remembering. *The Oxford Handbook of Metamemory*, 307.
- Finley, J. R., Tullis, J. G., & Benjamin, A. S. (2010). Metacognitive control of learning and remembering. In *New science of learning* (pp. 109–131). Springer.
- Fleming, S. M., & Daw, N. D. (2017). Self-evaluation of decision-making: A general bayesian framework for metacognitive computation. *Psychological Review, 124*(1), 91.
- Goldsmith, M., & Koriat, A. (2007). The strategic regulation of memory accuracy and informativeness. *Psychology of learning and motivation, 48*, 1–60.
- Kelley, C. M., & Sahakyan, L. (2003). Memory, monitoring, and control in the attainment of memory accuracy. *Journal of Memory and Language, 48*(4).
- Koriat, A., & Goldsmith, M. (1996). Monitoring and control processes in the strategic regulation of memory accuracy. *Psychological review, 103*(3), 490.
- Lagnado, D. A., & Sloman, S. (2004). The advantage of timely intervention. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 30*(4), 856.
- Lieder, F., & Griffiths, T. L. (2015). When to use which heuristic: A rational solution to the strategy selection problem. In *Cogsci*.
- Lieder, F., Plunkett, D., Hamrick, J. B., Russell, S. J., Hay, N., & Griffiths, T. (2014). Algorithm selection by rational metareasoning as a model of human strategy selection. In *Advances in neural information processing systems* (pp. 2870–2878).
- Little, R. J., & Rubin, D. B. (2014). *Statistical analysis with missing data*. John Wiley & Sons.
- Mezirow, J. (1981). A critical theory of adult learning and education. *Adult education quarterly, 32*(1), 3–24.
- Nelson, T. O., & Narens, L. (1990). The psychology of learning and motivation. *Metamemory: A theoretical framework and new findings*.
- Paris, S. G., & Paris, A. H. (2001). Classroom applications of research on self-regulated learning. *Educational psychologist, 36*(2), 89–101.
- Sobel, D. M., & Kushnir, T. (2006). The importance of decision making in causal learning from interventions. *Memory & Cognition, 34*(2), 411–419.
- Steyvers, M., Miller, B., Hemmer, P., & Lee, M. D. (2009). The wisdom of crowds in the recollection of order information. In *Advances in neural information processing systems* (pp. 1785–1793).
- Steyvers, M., Tenenbaum, J. B., Wagenmakers, E.-J., & Blum, B. (2003). Inferring causal networks from observations and interventions. *Cognitive science, 27*(3), 453–489.
- Surowiecki, J. (2004). *The wisdom of crowds: why the many are smarter than the few and how collective wisdom shapes business, economics, society and nations*. Little, Brown.
- Williams, R., et al. (2006). Generalized ordered logit/partial proportional odds models for ordinal dependent variables. *Stata Journal, 6*(1), 58.
- Winne, P. H., & Hadwin, A. F. (1998). Studying as self-regulated learning. *Metacognition in educational theory and practice, 93*, 27–30.
- Zimmerman, B. J. (1989). A social cognitive view of self-regulated academic learning. *Journal of educational psychology, 81*(3), 329.